Introduction to Simulink

CONTENTS

1 PREFACE

1.1 What is Simulink?

Simulink is a software package for modelling, simulating, and analyzing dynamic systems. It supports linear and nonlinear systems modelled in continuous time, sampled time, or a hybrid of the two. Systems can also be multirate, i.e. have different parts that are sampled or updated at different rates.

With Simulink, you can move beyond idealized linear models to explore more realistic nonlinear models, factoring in friction, air resistance, gear slippage, hard stops, and the other things that describe real-world phenomena. Simulink turns your computer into a lab for modelling and analyzing systems that simply wouldn't be possible or practical otherwise, whether the behaviour of an automotive clutch system, the flutter of an airplane wing, the dynamics of a predator-prey model, or the effect of the monetary supply on the economy.

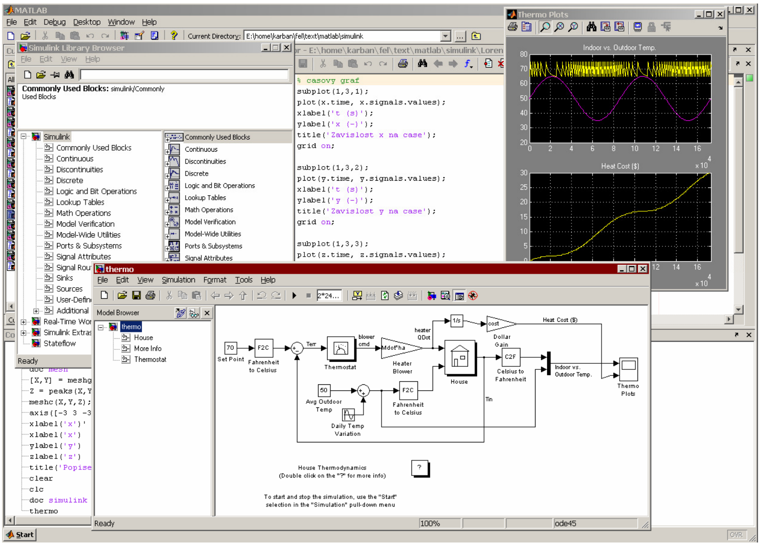


Figure 1 *Model in Simulink*

1.2 Dynamic systems

A dynamic system is in fact a way how to describe real objects and phenomena. It is possible to say, that the dynamic system is a set of variables and relations. Dynamic system can describe relations between variables and causality between variables as well. Variables describing an object or a phenomenon can be divided into to sets – parameters and state of system.

State of system is such a set of variables the knowledge of which and the input function, with the equation describing the dynamics, will provide the future state and output of the system.

1.2.1 Differential Equations

A dynamic system can be described by mean of set of the differential equations in form

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \cdots + a_1 y'(t) + a_0 y(t) =$$
$$= b_0 u(t) + b_1 u'(t) + \cdots + b_{n-1} u^{(n-1)}(t) + b_n u^{(n)}(t)$$

where a_i and b_i are constants, $u(t)$ is input parameter and $y(t)$ is output parameter. Continuous description of the system can be derived from common known physical laws (Maxwell's equations, Kirchhoff's laws, law of force balance, Newton's laws, etc.) and by elimination of all depended variables.

1.2.2 Transfer function

Transfer function of the continuous dynamic system is given as the rate of the Laplace transform of input and output variables.

$$F(p)=\frac{L\{y(t)\}}{L\{u(t)\}}=\left.\frac{Y(p)}{U(p)}\right|_{\text{zero initial conditions}}$$

System is described by following transfer function

$$F(p)=\frac{Y(p)}{U(p)}=\frac{3p+0,3}{2p^2+0,1p+2}.$$

After some rearrangement we will get

$$Y(p)\cdot\left(2p^2+0,1p+2\right)=U(p)\cdot\left(3p+0,3\right)$$

After transformation into time space we will get appropriate differential equation in form

$$2y''(t)+0,1y'(t)+2y(t)=3u'(t)+0,3u(t)$$

1.2.3 State of a system

The state of a system is described by a set of first-order differential equations in terms of the state variables $(x_1,\ x_2,\ \ldots,x_n)$.

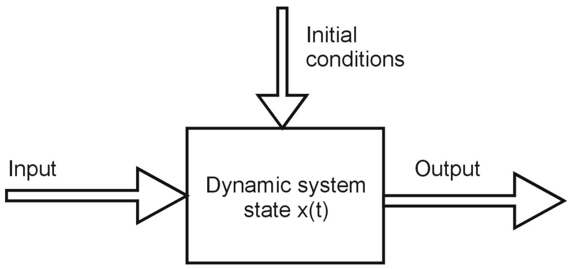


Figure 2 *Dynamic system*

This set of differential equations can be written in matrix form as follows:

$$\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{Bu}$$

The state differential equation relates the rate of change of the state of the system to the state of the system and the input signals. In general the output of dynamic system can be related to the state variables and the input signal by the output equation

$$\mathbf{y}=\mathbf{Cx}+\mathbf{Du},$$

where $\mathbf{y}$ is the set of output signals expressed in column vector form. It is possible to express the previous equations using block diagram.

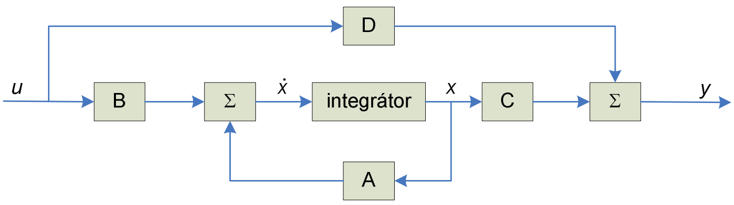


Figure 3 *Structural scheme of dynamic system*

A ... matrix of dynamic of the system
B ... input matrix
C ... output matrix
D ... matrix of direct acting input to output

A Simulink block diagram model is a graphical representation of a mathematical model of a dynamic system. The mathematical model of a dynamic system is described by a set of equations. The mathematical equations described by a block diagram model are known as algebraic, differential, and/or difference equations.

1.3 Preparing problems for simulating in Simulink

Simulate given electrical network in Simulink. Determine waveform of current through the inductor $i_{L1}(t)$.

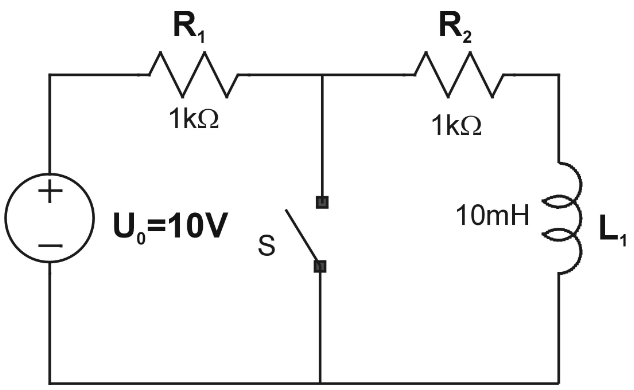


Figure 4 *First order dynamic system example*

Solution:

Given circuit can be described by the first order linear differential equation with constant coefficients.

$$R_2 i_L + L\frac{di_L}{dt} = 0 \quad \text{Initial condition: } i_L(0) = \frac{U_0}{R_1 + R_2}$$

after simple manipulation:

$$\frac{di_L}{dt} = -\frac{R_2}{L} i_L$$

It is possible to rewrite previous equation in the "matrix" form (There is only one differential equation in this case):

$$\dot{x}_1 = Ax_1 , \quad x(0) = \frac{U_0}{R_1 + R_2} = 5 \cdot 10^{-3}$$

$y = Cx_1$,

where $x_1 = i_L$, $A = -\frac{R_2}{L} = -10^5$ and $C = 1$

Now we can draw a block diagram based on first order differential equation mentioned.

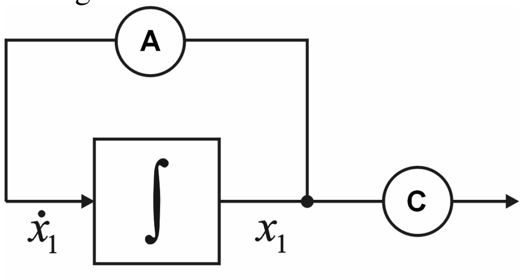


Figure 5 *Block diagram of simple dynamic system*

It is obvious fact that to each dynamic system representation it is possible to assign block diagram uniquely.

Simulink provides a graphical editor that allows you to create and connect instances of block types selected from libraries. Simulink provides libraries of blocks representing elementary system which can be used as building blocks.

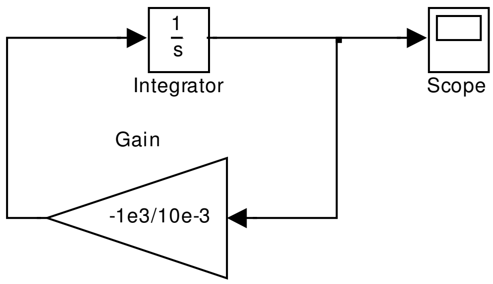


Figure 6 *Simulink model of simple dynamic system*

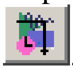
2 SIMULINK BASIC

2.1 Creating a model

2.1.1 Starting Simulink

There are two ways to initiate a Simulink session:

- Enter the `simulink` at the Matlab command prompt.

- Click on the Simulink icon on the Matlab toolbar .

On Microsoft Windows platforms, starting Simulink displays the Simulink Library Browser.

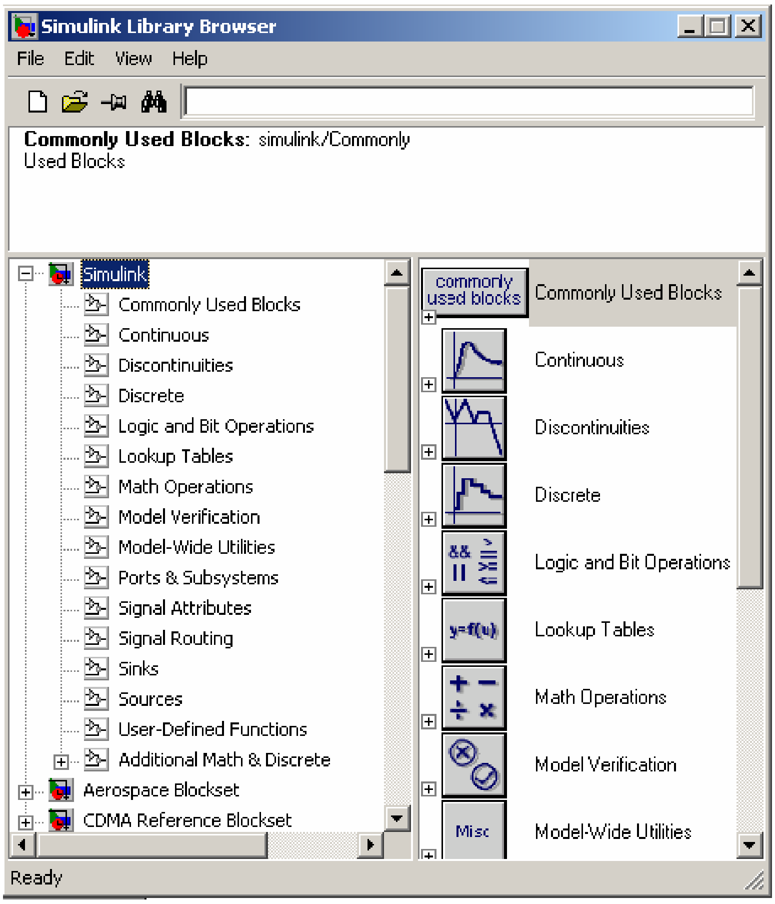


Figure 7 *Simulink Library Browser*

The Library Browser displays a tree-structured view of the Simulink block libraries installed on your system. You can build models by copying blocks from the Library Browser into a model window.

2.1.2 Creating a new model

Choose New from the library window's File menu and select Model (or press CTRL+N).

2.1.3 Placing objects

To create the model of electrical network form the previous chapter, you need to copy blocks into the model from the following Simulink block libraries:

- Sinks library (the Scope block)
- Continuous library (the Integrator block)
- Math operation library (the Gain block)

To copy the Integrator block form the Library Browser, first expand the Library Browser tree to display the blocks in the Continuous library blocks. Click the Integrator node to select the Integrator block and drag a copy of Integrator block in the model window.

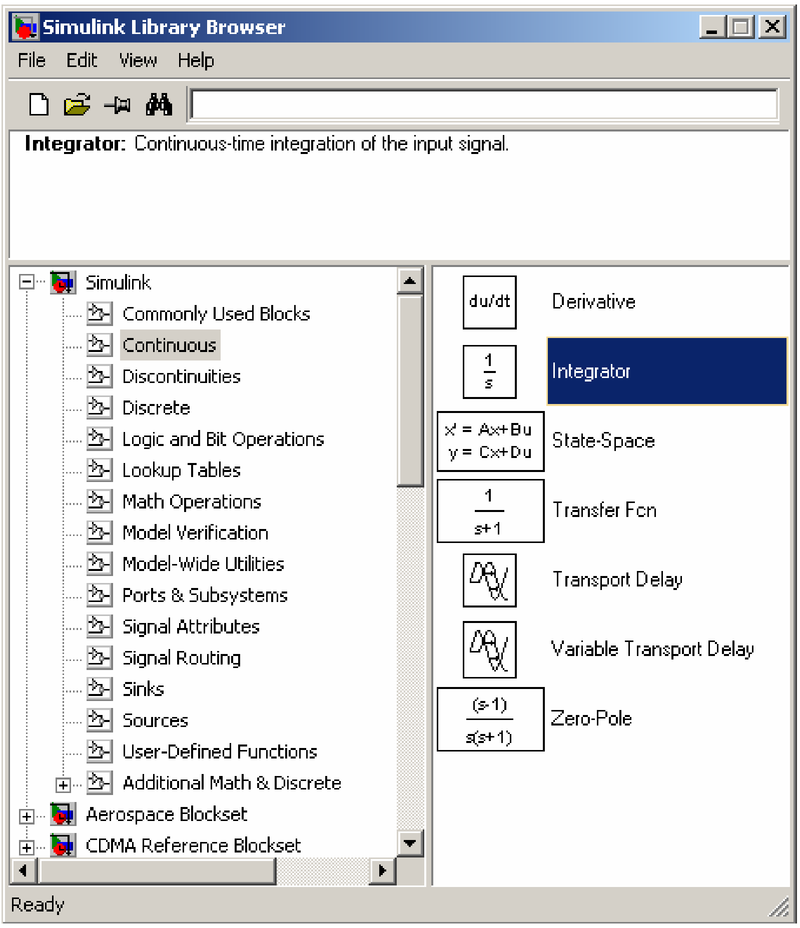


Figure 8 *Continuous library window*

Copy the rest of blocks in a similar way from their respective libraries into the model window. You can move a block from one place in the model window to another by dragging the block.

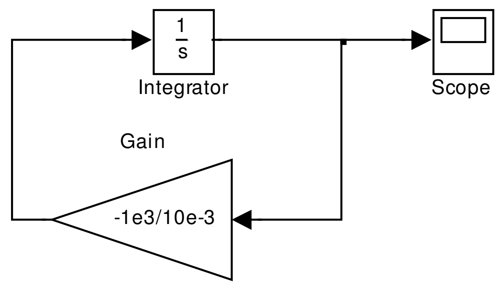


Figure 9 *Simulink model of simple dynamic system*

2.1.4 Connecting blocks

If you examine the blocks, you can see angle brackets on sides of the blocks. The > symbol pointing out of a block is an output port; if the symbol points to a block, it is an input block. If you want to connect blocks move the cursor to the port of the first block. Notice that the cursor shape changes to crosshairs. Hold down the mouse button and move the cursor to the port of the second block.

2.1.5 Setting up Simulink and Running simulation

First, open the Configuration Parameters dialog box by choosing Configuration Parameters from the Simulation menu. Type 5E−5 into the Stop time edit box. Close Configuration Parameters dialog box by clicking the OK button Simulink applies the parameters and closes the dialog box.

2.1.6 Simulation results displaying

Double-click the Scope block to open the display window. Then, choose Start from the Simulation menu and watch the simulation output on the Scope.

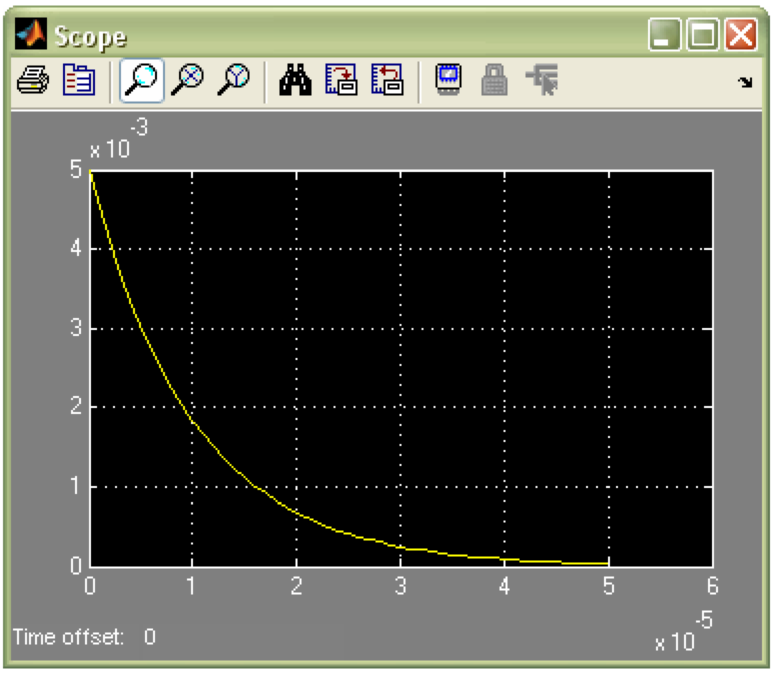


Figure 10 *Time evolution of current* i_L

3 EXAMPLES WITH SOLUTION

3.1 Electrical Circuit

3.1.1 Problem formulation

Simulate given circuit in Simulink and find waveforms of all currents in inductors and all voltages on capacitors. (Suppose zero initial conditions)

Given values:

$R_1 = 100\Omega,\quad R_2 = 120\Omega\ R_3 = 470\Omega,\quad C_1 = 47nF$

$C_2 = 200mF,\ C_3 = 1.2mF\ L_1 = 10mH,\ L_2 = 17mH\ ,\ U_0 = 10V$

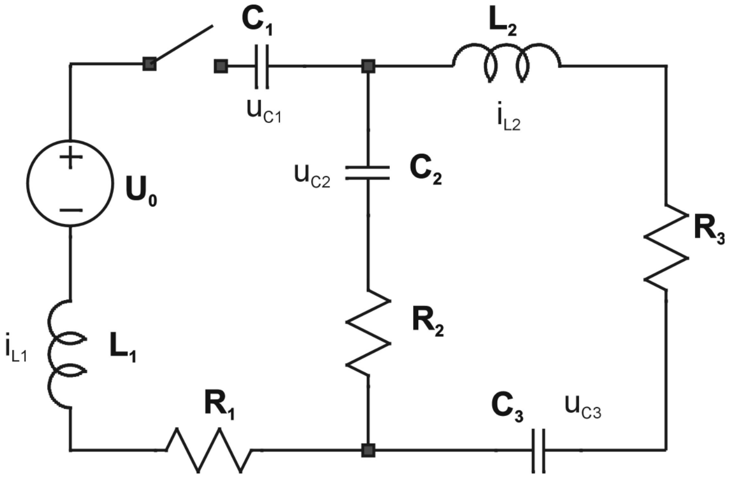


Figure 11 *Electrical network – 5th order dynamical system*

Given circuit can be described by following set of differential equations.

$$\frac{\mathrm{d}i_{L1}}{\mathrm{d}t} = -\frac{R_1 + R_2}{L_1} i_{L1} + \frac{R_2}{L_1} i_{L2} - \frac{1}{L_1} u_{C1} - \frac{1}{L_1} u_{C2} + \frac{U_0}{L_1}$$

$$\frac{\mathrm{d}i_{L2}}{\mathrm{d}t} = \frac{2R_1 + R_2}{L_2} i_{L1} - \frac{R_2 + R_3}{L_2} i_{L2} + \frac{1}{L_2} u_{C2} - \frac{1}{L_2} u_{C3}$$

$$\frac{\mathrm{d}u_{C1}}{\mathrm{d}t} = \frac{1}{C_1} i_{L1}$$

$$\frac{\mathrm{d}u_{C2}}{\mathrm{d}t} = \frac{1}{C_2} i_{L1} - \frac{1}{C_2} i_{L2}$$

$$\frac{\mathrm{d}u_{C3}}{\mathrm{d}t} = \frac{1}{C_3} i_{L2}$$

Define the state vector of the system representation:

$x_1 = i_{L1}$
$x_2 = i_{L2}$
$x_3 = u_{C1}$, then
$x_4 = u_{C2}$
$x_5 = u_{C3}$

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu}\\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du}\end{aligned}$$, where

$$\mathbf{A} = \begin{bmatrix} -\frac{R_1+R_2}{L_1} & \frac{R_2}{L_1} & -\frac{1}{L_1} & -\frac{1}{L_1} & 0 \\ \frac{2R_1+R_2}{L_2} & \frac{R_2+R_3}{L_2} & 0 & \frac{1}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 & 0 \\ \frac{1}{C_2} & \frac{-1}{C_2} & 0 & 0 & 0 \\ 0 & \frac{1}{C_3} & 0 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -22\cdot 10^3 & 12\cdot 10^3 & -100 & -100 & 0 \\ 188\cdot 10^3 & -34{,}7\cdot 10^3 & 0 & 58{,}8 & -58{,}8 \\ 21\cdot 10^6 & 0 & 0 & 0 & 0 \\ 5\cdot 10^3 & -5\cdot 10^3 & 0 & 0 & 0 \\ 0 & 833\cdot 10^3 & 0 & 0 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3.1.2 Creating Simulink model

1. Create new Model in Simulink.
2. You can build the model from basic blocks like Integrator or Gain similarly to the previous example, but simpler way is using State-Space block from Continuous library. Copy this block from the library into the model window.

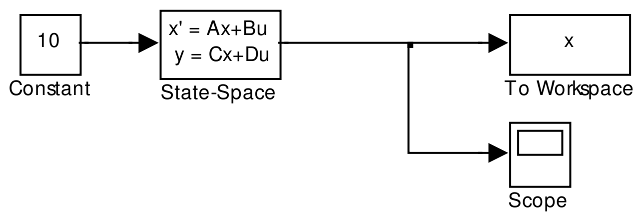


Figure 12 *Simulink model of 5th order dynamical system (electrical network).*

3. Double-click on the State-Space block **Function Block Parameters** dialog box appears.
4. Enter matrices of system representation into edit boxes, use standard Matlab convention. (see the picture bellow). Then click OK.

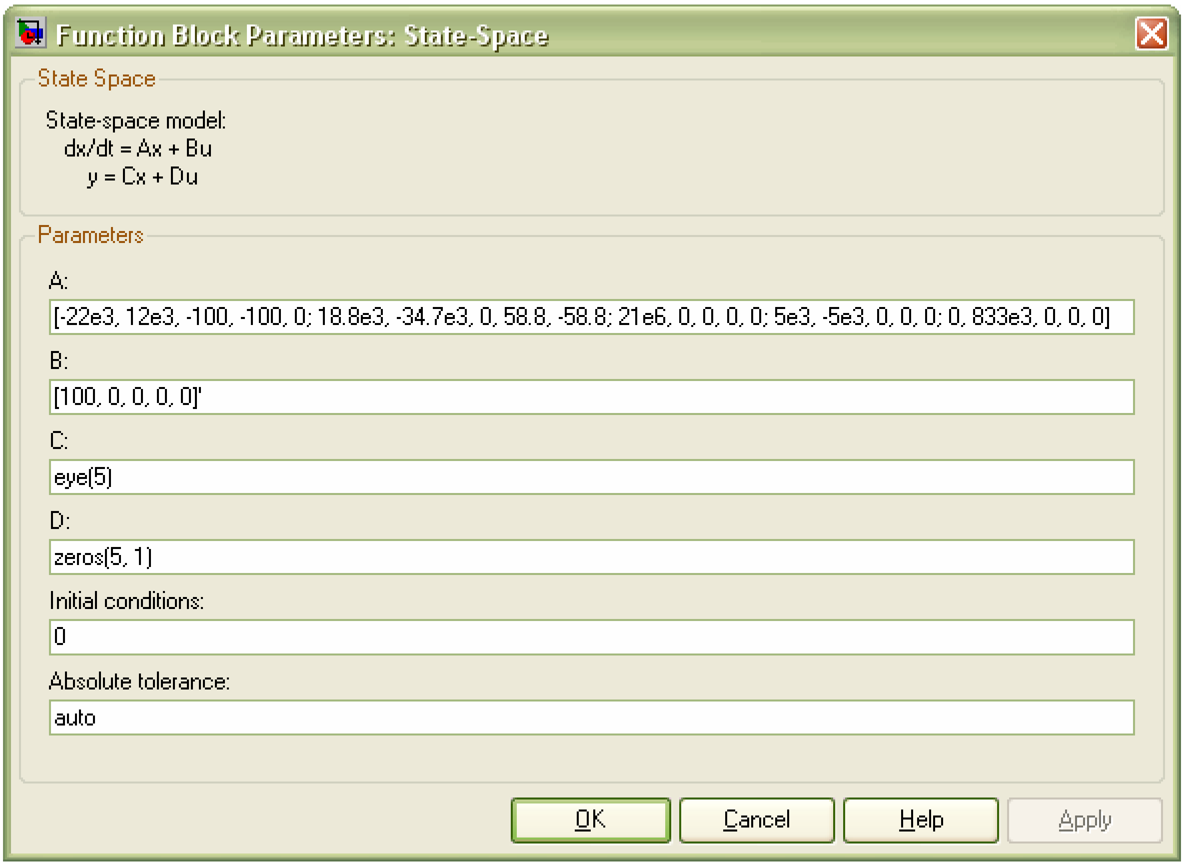


Figure 13 *Block detail*

5. Place the rest of blocks and connect them.
6. In the **Configuration Parameters** dialog box type `10E-4` into the **Stop time** edit box.
7. Double-click on the Scope block and then run simulation.
8. Click right on the scope window and chose **Autoscale** item.

3.2 Dynamic System described by set of differential equation

The dynamic system is described by following set of differential equation

$$\begin{aligned} \dot{x}_1 &= -x_1 + u \\ \dot{x}_2 &= -2x_2 + u \, . \\ y &= 6x_1 - 3x_2 \end{aligned}$$

Find time response on unit step (Dirac's impulse function) with width of two seconds and time response on sine wave with amplitude 1 and frequency 2 Hz.

We have the system with two input variables x_1 and x_2 and one output variable y. Matrix form of set of differential equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 6 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Unit step (Dirac's impulse function) is very useful for investigation of the response of dynamic system. It is given as an impulse with rectangular shape with width t_1 and its area equals one.

$$\delta(t) = \begin{cases} 0 & \text{pro}\, t \geq t_1 \\ \frac{1}{t_1} & \text{pro}\, 0 \leq t < t_1 \end{cases}$$

In Simulink we can use two blocks for step function `Step`.

For example the impulse starting in time 1 second and ending in time 3 seconds with amplitude 0,5 satisfy the definition of Dirac's impulse function. Its area is 0,5·(3-1) = 1.

Simulink model you can see on figure 14.

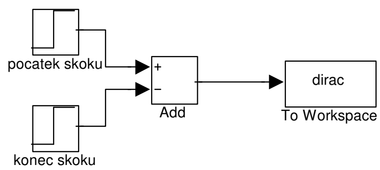


Figure 14 *Simulink model of unit impulse function*

Simulink blocks are shown on figure 15.

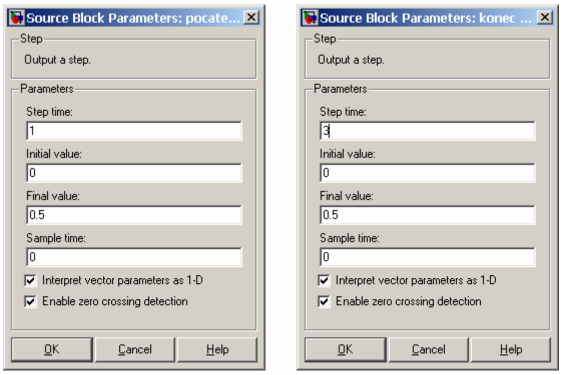


Figure 15 *Simulink blocks of unit impulse function*

In the `Configuration Parameters` dialog box set simulation time 6 seconds and maximal step 0.01 s. Time evolution of the impulse you can see on figure 16.

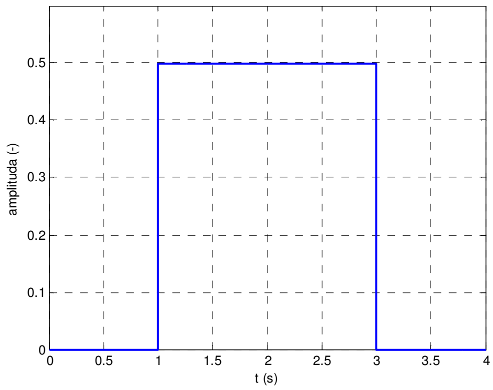


Figure 16. *Time evolution of the impulse*

For simulation of set of differential equation we use `State-Space` block enabling description of the system directly by matrices **A**, **B**, **C** and **D**.

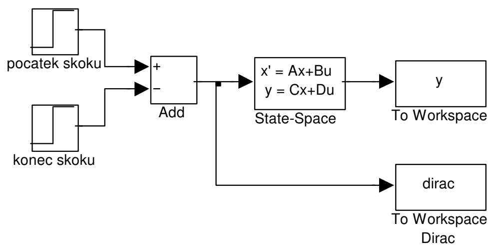


Figure 17. *Simulink model*

`State-Space` block is shown on figure 18.

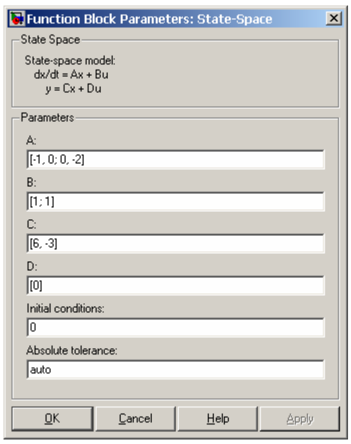


Figure 18 `State-Space` *block*

Run the simulation. Time response is shown on figure 19.

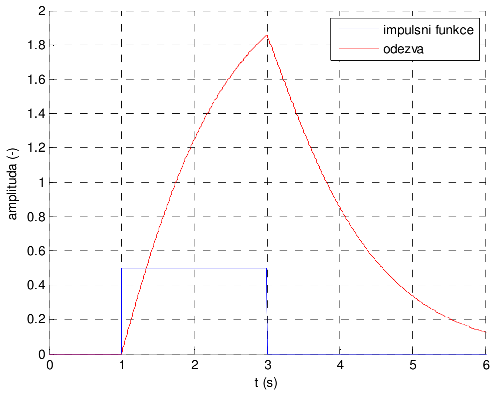


Figure 19 *Time response of unit step*

Time response of sine wave function we will investigate with substation unit step block and sine wave block (see figure 20)

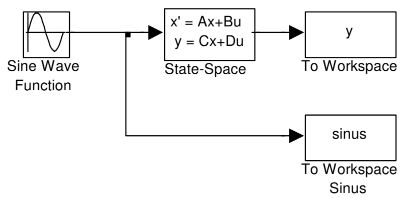


Figure 20 *Simulink model*

Time response is shown on figure 21.

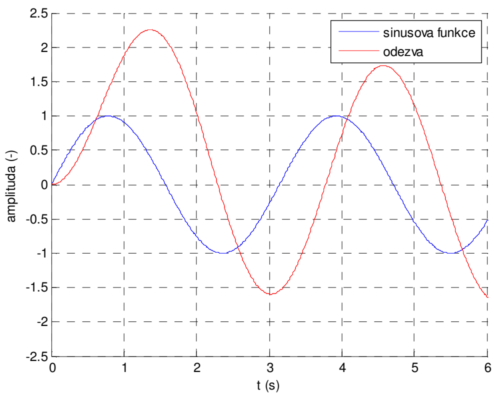


Figure 21 *Time response on sine wave function*

3.3 Bernard-Shaw Attractor

Bernard-Shaw Attractor is described by following set of differential equations

$$\dot{x}_1 = 10x_1\left(0,1 - x_2^2\right) + x_2$$
$$\dot{x}_2 = -x_1 + 0,5\sin(2,15t)$$

Create the model and simulate it in Simulink. Maximum step set to 0.05 s and stop time set to 500 s.

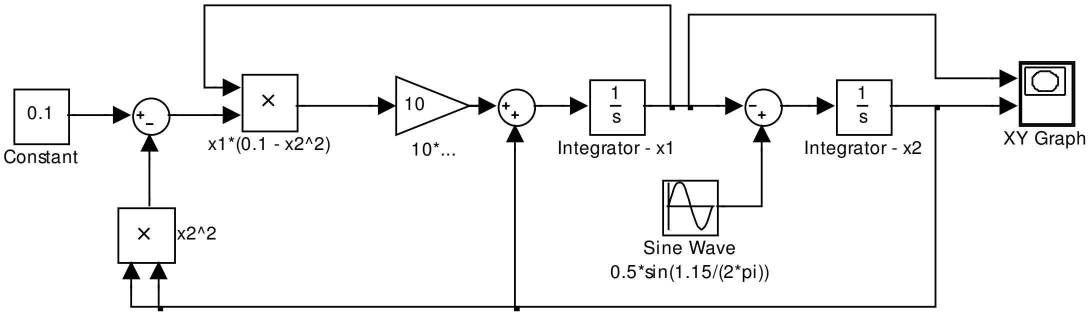


Figure 22 *Simulink model*

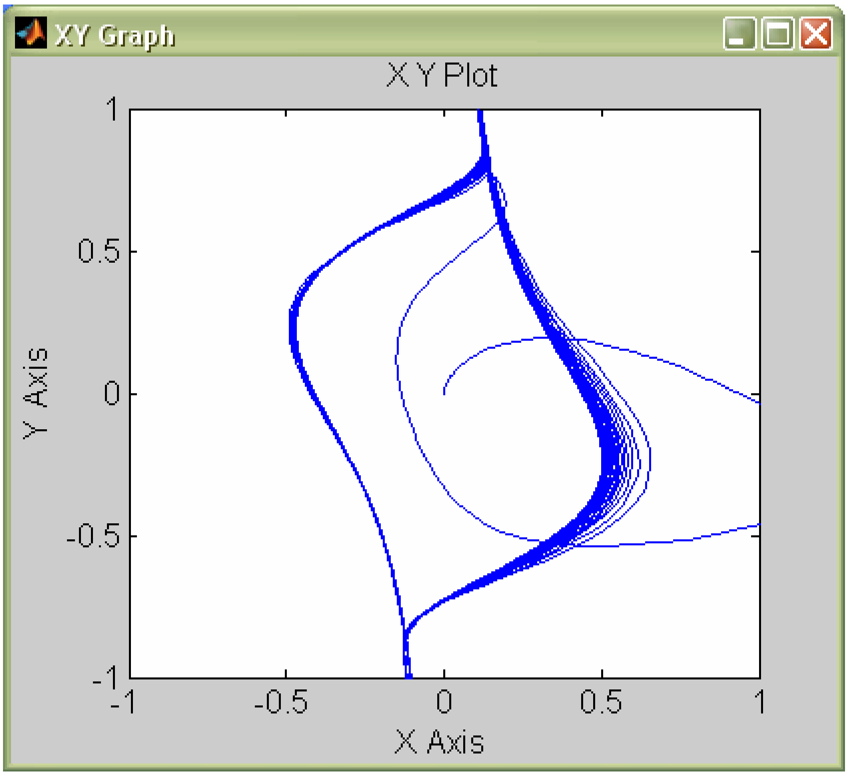


Figure 23 *Results*

3.4 DC Motor

Create model of DC motor with separate excitation. Inductance of the armature $L = 0{,}1$ H, resistance of the armature $R = 2\ \Omega$, voltage $U = 110$ V, load torque $m_z = 0{,}02$ Nm, moment of inertia $J = 0{,}1$ kg/m^2 and electro-mechanical constant of the motor $k = 0{,}3$ Vs. Create it as separate block (subsystem) with one input (voltage) and two outputs (angular velocity and current).

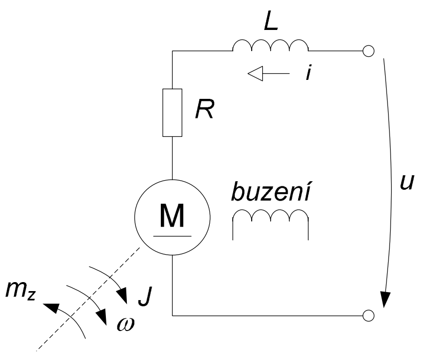


Figure 24 *DC Motor*

Set of differential equations describing DC motor with separate excitation are given by electrical balance in circuit of the armature.

$$U = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + k\omega$$

and mechanical balance in the rotor.

$$ki = J\frac{\mathrm{d}\omega}{\mathrm{d}t} + m_z \,.$$

After some rearrangement we will get set of equations in form

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{U}{L} - \frac{R}{L}i - \frac{k}{L}\omega$$

and

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{k}{J}i - \frac{m_z}{J} \,.$$

Firstly we will create block of the DC motor with one input (circle in left side of the figure) and two outputs (circles in right side of the figure). Subsystem can be created by selecting appropriate blocks and choosing *Create subsystem* in context menu.

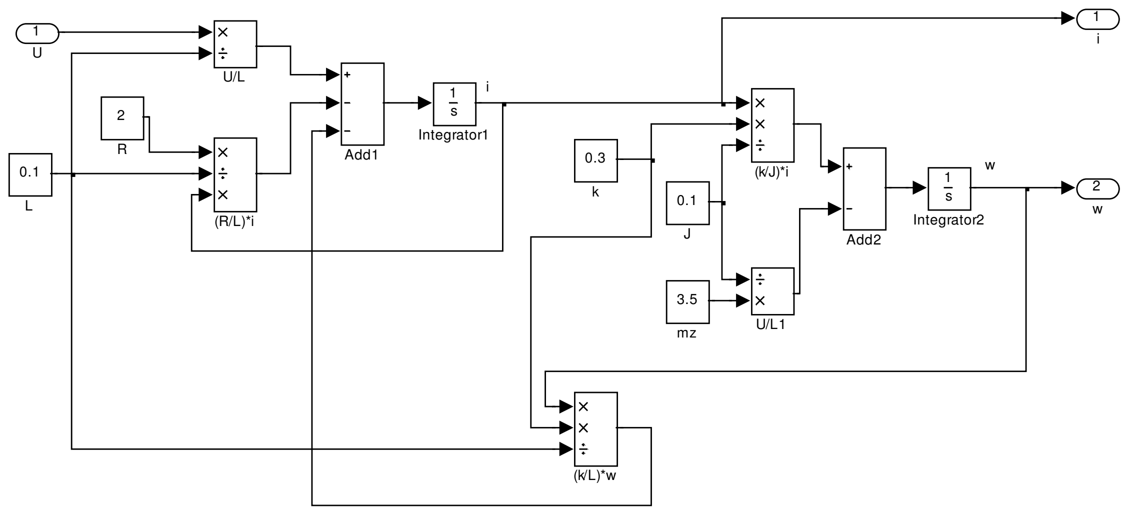


Figure 25 *Simulink model – detail*

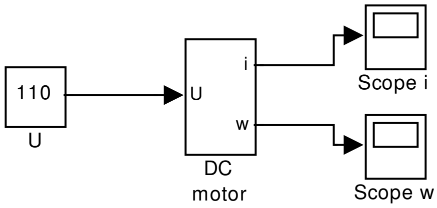


Figure 26 *Simulink model*

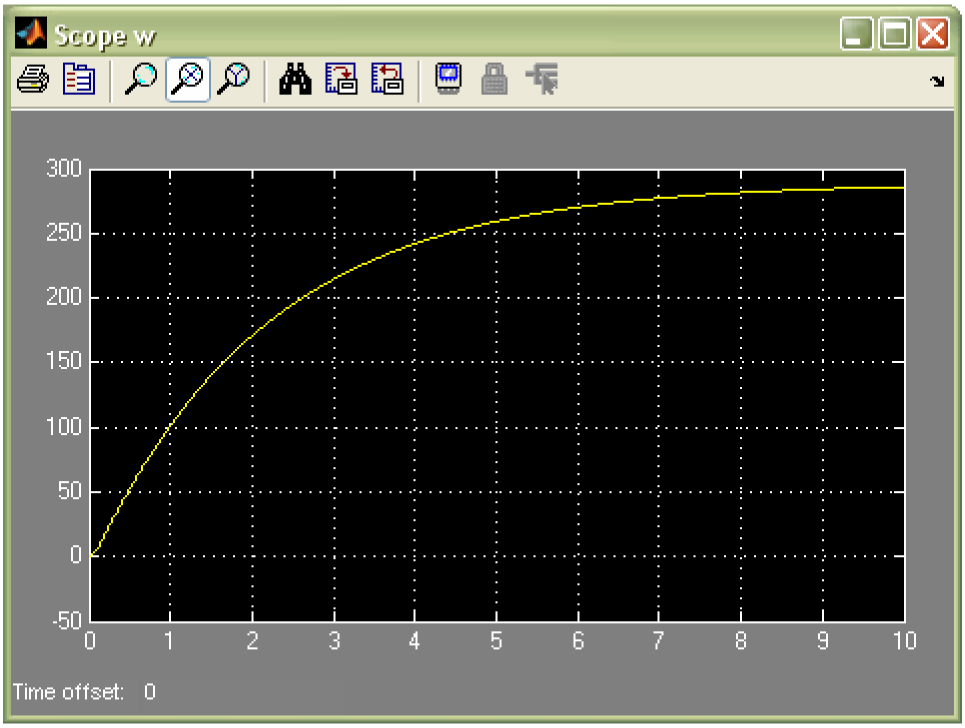


Figure 27 *Angular velocity*

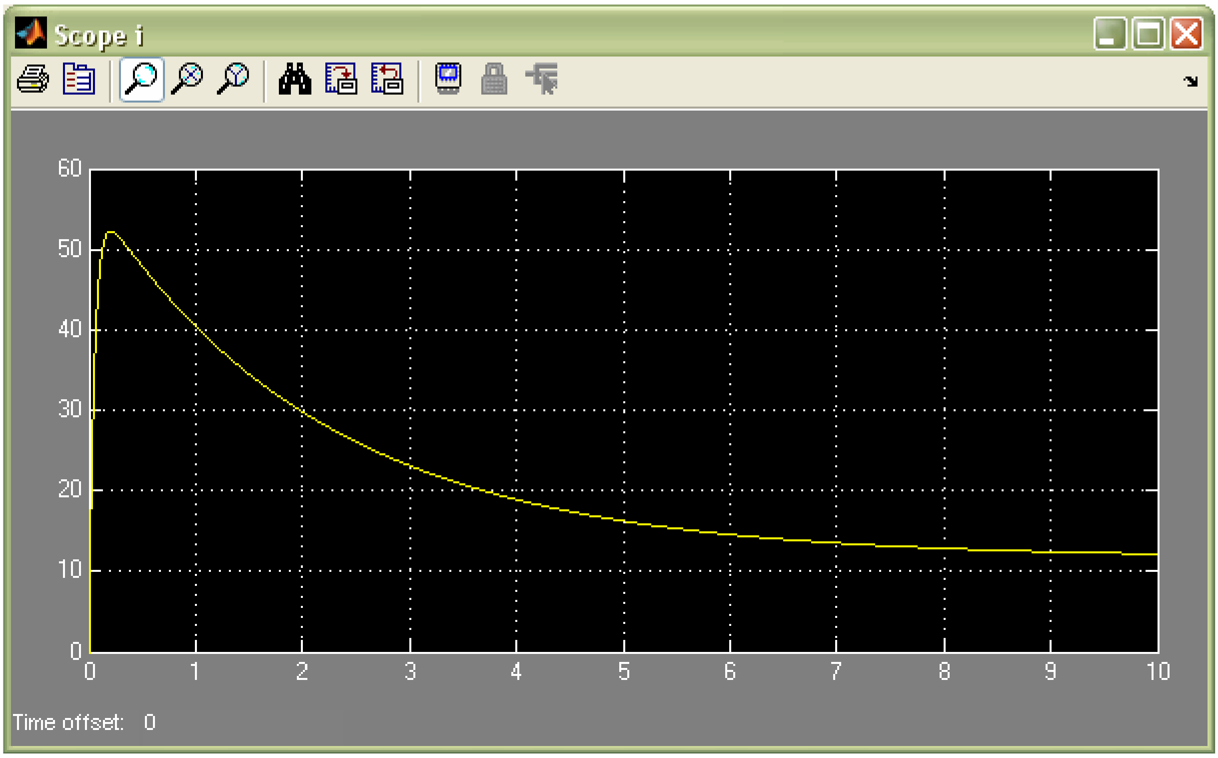


Figure 28 *Current carrying through the armature*

4 EXERCISES

1. Suppose electrical network from example in chapter 3.1.1, where the constant voltage source is replaced by a harmonic voltage source. Create Simulink model of this electrical network and simulate it.

2. Simulink libraries include blocks for mathematical operations. Try to find proper blocks and draw a graph of following functions :
   a. $f(x) = e^x \qquad \forall x \in \langle 0, 3 \rangle$
   b. $f(x) = 5x^2 - 2x + 1 \qquad \forall x \in \langle -2, 2 \rangle$

3. In Simulink it is possible to describe a dynamical system using its transfer function. Try to find a unit step response of dynamical system using proper Simulink blocks. The transfer function of the dynamical system is

$$F(s) = \frac{5}{s^2 + 2s + 25}$$

4. The Lorenz attractor is a chaotic map, noted for its butterfly shape. The map shows how the state of a dynamical system (the three variables of a three-dimensional system) evolves over time in a complex, non-repeating pattern, often described as beautiful. The attractor itself, and the equations from which it is derived, were introduced by Edward Lorenz in 1963, who derived it from the simplified equations of convection rolls arising in the equations of the atmosphere. From a technical standpoint, the system is nonlinear, three-dimensional and deterministic. In 2001 it was proven by W. Tucker that for a certain set of parameters the system exhibits chaotic behavior and displays what is today called a strange attractor. The strange attractor in this case is a fractal of Hausdorff dimension between 2 and 3. Grassberger (1983) has estimated the Hausdorff dimension to be 2.06 ± 0.01 and the correlation dimension to be 2.05 ± 0.01. The equations that govern the Lorenz attractor are:

$$\frac{dx}{dt} = \sigma(y - x),$$

$$\frac{dy}{dt} = x(r - z) - y,$$

$$\frac{dz}{dt} = xy - bz$$

where σ is called the Prandtl number and r is called the Rayleigh number. All σ, r, $b > 0$, but usually $\sigma = 10$, $b = 8/3$ and r is varied. The system is stable for $r = 10$ and chaotic for $r = 28$. Try to simulate the problem. Computation time choose 200 s for stable system and 15 s for chaotic system, maximal step choose 0.005 s. Initial conditions are $x_0 = 20$, $y_0 = 5$ and $z_0 = -5$. Export values of variables x, y and z to Matlab workspace and plot a graph using plot3 function.